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The scattering of harmonic elastic anti-plane shear waves by a Griffith crack in a piezoelectric material plane by using the non-local theory

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Abstract

In this paper, the dynamic behavior of a Griffith crack in a piezoelectric material plane under anti-plane shear waves is investigated by using the non-local theory for impermeable crack face conditions. For overcoming the mathematical difficulties, a one-dimensional non-local kernel is used instead of a two-dimensional one for the anti-plane dynamic problem to obtain the stress and the electric displacement near the crack tips. By using the Fourier transform, the problem can be solved with the help of two pairs of dual integral equations. These equations are solved using the Schmidt method. Contrary to the classical elasticity solution, it is found that no stress and electric displacement singularity is present near the crack tip. The non-local dynamic elastic solutions yield a finite hoop stress near the crack tip, thus allowing for a fracture criterion based on the maximum dynamic stress hypothesis. The finite hoop stress at the crack tip depends on the crack length, the circular frequency of incident wave and the lattice parameter. For comparison results between the non-local theory and the local theory for this problem, the same problem in the piezoelectric materials is also solved by using local theory. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Elastic waves; Piezoelectric materials; Non-local theory; Fourier integral transform; Crack; Schmidt method

1. Introduction

It is well known that piezoelectric materials produce an electric field when deformed, and undergo deformation when subjected to an electric field. The coupling nature of piezoelectric

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materials has attracted wide applications in electric–mechanical and electric devices, such as electric–mechanical actuators, sensors and structures. When subjected to mechanical and electrical loads in service, these piezoelectric materials can fail prematurely due to their brittleness and presence of defects or flaws produced during their manufacturing process. Therefore, it is important to study the electro-elastic interaction and fracture behavior of piezoelectric materials.

Many studies have been made on the electro-elastic fracture mechanics based on the modeling and analyzing of one crack in the piezoelectric materials (see, for example, [3,12,18,19,22,23,25,29, 30,32,33]). The problem of the interacting fields among multiple cracks in a piezoelectric material has been studied by Han [14]. In Han's paper, the crack is treated as continuously distributed dislocations with the density function to be determined according to the conditions of external loads and crack surface. Most recently, Chen and Karihaloo [2] considered an infinite piezoelectric ceramic with impermeable crack-face boundary condition under arbitrary electro-mechanical impact. Sosa and Khutoryansky [26] investigated the response of piezoelectric bodies disturbed by internal electric sources. The impermeable boundary condition on the crack surface was widely used in the works [2,22,23,28,29]. However, these solutions contain stress and electric displacement singularity. This is not reasonable according to the physical nature. For overcoming the stress singularity in the classical elastic theory, Eringen [6,8,9] used the non-local theory to discuss the state of stress near the tip of a sharp line crack in an elastic plane subject to uniform tension, shear and anti-plane shear. Zhou [34–37] used the non-local theory to study the state of the dynamic stress near the tip of a line crack or two line cracks in an elastic plane. These solutions did not contain any stress singularity, thus resolving a fundamental problem that persisted over many years. This enables us to employ the maximum stress hypothesis to deal with fracture problems in a natural way.

In the present paper, the scattering of harmonic elastic anti-plane shear waves by a Griffith impermeable crack subjects to anti-plane shear in piezoelectric materials is investigated by using the non-local theory. The traditional concept of linear elastic fracture mechanics and the non-local theory are extended to include the piezoelectric effects. For overcoming the mathematical difficulties, one-dimensional non-local kernel function is used instead of two-dimensional kernel function for the anti-plane dynamic problem to obtain the stress and electric displacement occur at the crack tips. For obtaining the theoretical solution and discussing the probability of using the non-local theory to solve the dynamic fracture problem in the piezoelectric materials, one has to accept some assumptions as Nowinski's [20,21]. Certainly, the assumption should be further investigated to satisfy the realistic condition. Fourier transform is applied and a mixed boundary value problem is reduced to two pairs of dual integral equations. In solving the dual integral equations, the crack surface displacement and electric potential are expanded in a series of Jacobi polynomials. This process is quite different from that adopted in previous works [3,6,8,9,12,14,22,23,25,29,30,32,33]. As expected, the solution in this paper does not contain the stress and electric displacement singularity at the crack tip, thus clearly indicating the physical nature of the problem, namely, in the vicinity of the geometrical discontinuities in the body, the non-local intermolecular forces are dominant. For such problems, therefore, one must resort to theories incorporating non-local effects, at least in the neighborhood of the discontinuities.

2. Basic equations of non-local piezoelectric materials

For the anti-plane shear problem, the basic equations of linear, homogeneous, isotropic, non-local piezoelectric materials, with vanishing body force are (see e.g. [9,24,34,37]):

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} = \rho \frac{\partial^2 w}{\partial t^2}, \tag{1}$$

$$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} = 0, \tag{2}$$

$$\tau_{kz}(X,t) = \int_V [c'_{44}(|X'-X|) w_{,k}(X',t) + e'_{15}(|X'-X|)\phi_{,k}(X',t)]\,\mathrm{d}V(X') \quad (k = x, y), \tag{3}$$

$$D_k(X,t) = \int_V [e'_{15}(|X'-X|) w_{,k}(X',t) - \varepsilon'_{11}(|X'-X|)\phi_{,k}(X',t)]\,\mathrm{d}V(X') \quad (k = x, y), \tag{4}$$

where the only difference from classical elastic theory and the piezoelectric theory is in the stress and the electric displacement constitutive equations (3) and (4) in which the stress $\tau_{zk}(X,t)$ and the electric displacement $D_k(X,t)$ at a point X depends on $w_{,k}(X,t)$ and $\phi_{,k}(X,t)$, at all points of the body. w and ϕ are the mechanical displacement and electric potential. ρ is the mass density of the piezoelectric materials. For homogeneous and isotropic piezoelectric materials there exist only three material parameters, $c'_{44}(|X'-X|)$, $e'_{15}(|X'-X|)$ and $\varepsilon'_{11}(|X'-X|)$ which are functions of the distance $|X'-X|$. The integrals in Eqs. (3) and (4) are over the volume V of the body enclosed within a surface ∂V. As discussed in the papers (see e.g. [5,7]), it can be assumed in the form of $c'_{44}(|X'-X|)$, $e'_{15}(|X'-X|)$ and $\varepsilon'_{11}(|X'-X|)$ for which the dispersion curves of plane elastic waves coincide with those known in lattice dynamics. Among several possible curves the following has been found to be very useful:

$$(c'_{44}, e'_{15}, \varepsilon'_{11}) = (c_{44}, e_{15}, \varepsilon_{11})\alpha(|X'-X|), \tag{5}$$

where $\alpha(|X'-X|)$ is known as the influence function, and is the functions of the distance $|X'-X|$. $c_{44}, e_{15}, \varepsilon_{11}$ are the shear modulus, piezoelectric coefficient and dielectric parameter, respectively. Substitution of Eq. (5) into Eqs. (3) and (4) yields

$$\tau_{kz}(X,t) = \int_V \alpha(|X'-X|)\sigma_{kz}(X',t)\,\mathrm{d}V(X') \quad (k = x, y), \tag{6}$$

$$D_k(X,t) = \int_V \alpha(|X'-X|) D_k^c(X',t)\,\mathrm{d}V(X') \quad (k = x, y), \tag{7}$$

where

$$\sigma_{kz} = c_{44}\, w_{,k} + e_{15}\phi_{,k}, \tag{8}$$

$$D_k^c = e_{15}\, w_{,k} - \varepsilon_{11}\phi_{,k}. \tag{9}$$

Expressions (8) and (9) are the classical constitutive equations.

3. The crack model

It is assumed that there is a Griffith crack of length $2l$ along the x-axis in a piezoelectric material plane as shown in Fig. 1. Let ω be the circular frequency of the incident wave. $-\tau_0$ is a magnitude of the incident wave. In what follows, the time dependence of all field quantities assumed to be of the form $e^{-i\omega t}$ will be suppressed but understood. It is further supposed that the two faces of the crack do not come in contact during vibrations. The piezoelectric boundary-value problem for anti-plane shear is considerably simplified if we consider only the out-of-plane displacement and the in-plane electric fields. When the cracks are subjected to the harmonic elastic waves and a constant electric displacement $D_y = -D_0$, as discussed by Srivastava [27], Yu [31] and Eringen [9] papers, the boundary conditions on the crack faces at $y = 0$ are (in this paper, we just consider the perturbation stress field and the perturbation electric displacement field):

$$\tau_{yz}(x,0,t) = -\tau_0, \quad |x| \leqslant l, \tag{10}$$

$$D_y(x,0,t) = -D_0, \quad |x| \leqslant l, \tag{11}$$

$$w(x,0,t) = \phi(x,0,t) = 0, \quad |x| > l, \tag{12}$$

$$w(x,y,t) = \phi(x,y,t) = 0 \quad \text{for } (x^2+y^2)^{1/2} \to \infty. \tag{13}$$

Substituting Eqs. (6) and (7) into Eqs. (1) and (2), respectively, using Green–Gauss theorem, it can be obtained (see e.g. [9]):

$$\int\int_V \alpha(|x'-x|,|y'-y|)\left[c_{44}\nabla^2 w(x',y',t) + e_{15}\nabla^2\phi(x',y',t)\right]\mathrm{d}x'\mathrm{d}y' - \int_{-l}^{l} \alpha(|x'-x|,0)[\![\sigma_{yz}(x',0,t)]\!]\mathrm{d}x' = \rho\frac{\partial^2 w}{\partial t^2}, \tag{14}$$

$$\int\int_V \alpha(|x'-x|,|y'-y|)\left[e_{15}\nabla^2 w(x',y',t) - \varepsilon_{11}\nabla^2\phi(x',y',t)\right]\mathrm{d}x'\mathrm{d}y' - \int_{-l}^{l} \alpha(|x'-x|,0)[\![D_y^c(x',0,t)]\!]\mathrm{d}x' = 0, \tag{15}$$

where the boldface bracket indicates a jump at the crack line. $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ is the two-dimensional Laplace operator. Because of the assumed symmetry in geometry and loading, it is

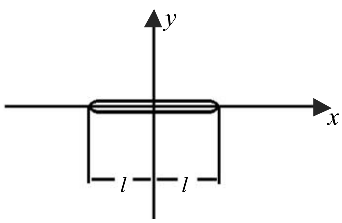

Fig. 1. Crack in a piezoelectric material body.

sufficient to consider the problem for $0 \leqslant x \leqslant \infty$, $0 \leqslant y \leqslant \infty$ only. Under the applied anti-plane shear load on the unopened surfaces of the crack, the displacement field and the electric displacement possess the following symmetry regulations:

$$w(x,-y,t) = -w(x,y,t), \quad \phi(x,-y,t) = -\phi(x,y,t). \tag{16}$$

Using Eq. (16), we find that

$$[\![\sigma_{yz}(x,0,t)]\!] = 0, \tag{17}$$

$$[\![D_y^c(x,0,t)]\!] = 0. \tag{18}$$

Hence the line integrals in Eqs. (14) and (15) vanish. By taking the Fourier transform of (14) and (15) with respect to x', it can be shown that

$$\int_0^\infty \bar{\alpha}(|s|,\ |y'-y|)\left\{c_{44}\left[\frac{\mathrm{d}^2\bar{w}(s,y',t)}{\mathrm{d}y^2} - s^2\bar{w}(s,y',t)\right] + e_{15}\left[\frac{\mathrm{d}^2\bar{\phi}(s,y',t)}{\mathrm{d}y^2} - s^2\bar{\phi}(s,y',t)\right]\right\}\mathrm{d}y' = -\rho\omega^2\bar{w}, \tag{19}$$

$$\int_0^\infty \bar{\alpha}(|s|,\ |y'-y|)\left\{e_{15}\left[\frac{\mathrm{d}^2\bar{w}(s,y',t)}{\mathrm{d}y^2} - s^2\bar{w}(s,y',t)\right] - \varepsilon_{11}\left[\frac{\mathrm{d}^2\bar{\phi}(s,y',t)}{\mathrm{d}y^2} - s^2\bar{\phi}(s,y',t)\right]\right\}\mathrm{d}y' = 0. \tag{20}$$

Here a superposed bar indicates the Fourier transform, e.g.

$$\bar{f}(s,y) = \int_0^\infty f(x,y)\exp(\mathrm{i}sx)\,\mathrm{d}x.$$

What now remains is to solve the integrodifferential equations (19) and (20) for the function w and ϕ. It seems obvious that a rigorous solution of such a problem encounters serious if not unsurmountable mathematical difficulties, and one has to resort to an approximate procedure. In the given problem, according to the Nowinski's [20,21] papers, the appropriate numerical procedure seems to follow naturally from the hypothesis of the attenuating neighborhood underlying the theory of non-local continua. According to this hypothesis, the influence of the particle of the body, on the thermoelectric state at the particle under observation, subsides rather rapidly with an increasing distance from particle. In the classical theory, the function that characterizes the particle interactions is the Dirac delta function since in this theory the actions are assumed to have a zero range. In non-local theories the intermolecular forces may be represented by a variety of functions as long as their values decrease rapidly with the distance. In the present study, as adequate functions it was decided to select the terms, $\delta_n(y'-y)$, $n = 1,2,\ldots,$ of the so-called δ-sequences. A δ-sequence, as generally known, converges to the (in the present case a one-dimensional) Dirac delta function, $\delta(y'-y)$. With respect to the terms of the adopted delta sequence, it was accepted the following simplifying assumptions: (See [20,21]. Nowinski had solved several non-local problems by using assumption of this kind.)

(a) For a sufficiently large j (as compared with the sphere of interactions of the particles), it is permissible to make the replacement

$$\int_{-j}^{j} f(y')\delta_n(y'-y)\,\mathrm{d}y' \approx \int_{-\infty}^{\infty} f(y')\delta(y'-y)\,\mathrm{d}y'. \tag{21}$$

(b) As a consequence, the terms $\delta_n(y'-y), n\rangle\rangle 1$, acquire the shifting property of the Dirac function,

$$\int_{-j}^{j} f(y')\delta_n(y'-y)\,\mathrm{d}y' \approx f(y). \tag{22}$$

The influence function was sought in the separable form. So according to the above discussion, the non-local interaction in y direction can be ignored. In view of our assumptions, it can be given

$$\bar{\alpha}(|s|,|y'-y|) = \bar{\alpha}_0(s)\delta_n(y'-y). \tag{23}$$

From Eqs. (19) and (20), it can be shown that

$$\bar{\alpha}_0(s)\left\{c_{44}\left[\frac{\mathrm{d}^2\bar{w}(s,y,t)}{\mathrm{d}y^2} - s^2\bar{w}(s,y,t)\right] + e_{15}\left[\frac{\mathrm{d}^2\bar{\phi}(s,y,t)}{\mathrm{d}y^2} - s^2\bar{\phi}(s,y,t)\right]\right\} = -\rho\omega^2\bar{w}, \tag{24}$$

$$e_{15}\left[\frac{\mathrm{d}^2\bar{w}(s,y,t)}{\mathrm{d}y^2} - s^2\bar{w}(s,y,t)\right] - \varepsilon_{11}\left[\frac{\mathrm{d}^2\bar{\phi}(s,y)}{\mathrm{d}y^2} - s^2\bar{\phi}(s,y)\right] = 0. \tag{25}$$

Because of symmetry, it suffices to consider the problem in the first quadrant only. The solution of Eqs. (24) and (25) does not present difficulties, it can be written as follows, respectively ($y \geqslant 0$):

$$w(x,y,t) = \frac{2}{\pi}\int_0^{\infty} A(s)\mathrm{e}^{-\gamma y}\cos(xs)\,\mathrm{d}s,\ \phi(x,y,t) - \frac{e_{15}}{\varepsilon_{11}}w(x,y,t) = \frac{2}{\pi}\int_0^{\infty} B(s)\mathrm{e}^{-sy}\cos(xs)\,\mathrm{d}s, \tag{26}$$

where $\gamma^2 = s^2 - \omega^2/c^2\bar{\alpha}_0(s)$, $c^2 = \mu/\rho, \mu = c_{44} + \frac{e_{15}^2}{\varepsilon_{11}}$. $A(s), B(s)$ are to be determined from the boundary conditions.

According to the boundary conditions (10)–(12), it can be obtained

$$\frac{2}{\pi}\int_0^{\infty} \bar{\alpha}_0(s)\gamma A(s)\cos(sx)\,\mathrm{d}s = \frac{1}{\mu}\left(\tau_0 + \frac{e_{15}D_0}{\varepsilon_{11}}\right), \quad |x| \leqslant l, \tag{27}$$

$$\frac{2}{\pi}\int_0^{\infty} A(s)\cos(sx)\,\mathrm{d}s = 0, \quad |x| > l \tag{28}$$

and

$$\frac{2}{\pi}\int_0^\infty \bar{\alpha}_0(s)sB(s)\cos(sx)\,\mathrm{d}s = -\frac{D_0}{\varepsilon_{11}}, \quad |x| \leqslant l, \tag{29}$$

$$\frac{2}{\pi}\int_0^\infty B(s)\cos(sx)\,\mathrm{d}s = 0, \quad |x| > l. \tag{30}$$

Eqs. (27)–(30) are the dual integral equations of this problem.

4. Solution of the dual integral equation

The non-local function α will depend on a characteristic length ratio a/l, where a is an internal characteristic length (e.g., lattice parameter, granular distance. In this paper, a represents lattice parameter.) and l is an external characteristic length (e.g., crack length, wave-length. In this paper, l represents the crack length). By matching the dispersion curves of plane waves with those of atomic lattice dynamics (or experiments), we can determine the non-local modulus function α for given material.Here, the only difference between the classical and non-local equations is in the introduction of the function $\bar{\alpha}_0(s)$, it is logical to utilize the classical solution to convert the system Eqs. (27)–(30) to an integral equation of the second kind which is generally better behaved. If $\bar{\alpha}_0(s) = 1$ (the classical elastic case), Eqs. (27)–(30) reduce to the dual integral equations for the same problem in classical elasticity. Of course, the dual integral equations (27)–(30) can be considered to be a single integral equation of the first kind with a discontinuous kernel [8]. It is well known in the literature that integral equations of the first kind are generally ill-posed in the sense of Hadamard, e.g. small perturbations of the data can yield arbitrarily large changes in the solution. This makes the numerical solution of such equations quite difficult. In this paper, Schmidt method [17] was used to overcome the difficulty. As discussed by Eringen's [6,8–10] and Nowinski's [20,21] papers, it was taken

$$\alpha_0 = \chi_0 \exp\Big(-(\beta/a)^2(x'-x)^2\Big), \tag{31}$$

$$\chi_0 = \beta/a\sqrt{\pi}, \tag{32}$$

where β is a constant (here β is a constant appropriate to each material.). a is the lattice parameter. So it can be obtained

$$\bar{\alpha}_0(s) = \exp\Big(-(sa)^2/(2\beta)^2\Big) \tag{33}$$

and $\bar{\alpha}_0(s) = 1$ for the limit $a \to 0$, so that Eqs. (27)–(30) reduce to the well-known equation of the classical theory. Here the Schmidt method can be used to solve the dual integral equations (27)–(30). The displacement w and the electric potential ϕ can be represented by the following series:

$$w(x,0,t) = \sum_{n=1}^{\infty} a_n P_{2n-2}^{(1/2,1/2)}\left(\frac{x}{l}\right)\left(1-\frac{x^2}{l^2}\right)^{1/2} \quad \text{for } -l \leqslant x \leqslant l, \ \ y=0, \tag{34}$$

$$w(x,0,t) = 0 \quad \text{for } |x| > l, \ \ y=0, \tag{35}$$

$$\phi(x,0,t)=\sum_{n=1}^{\infty} b_n P_{2n-2}^{(1/2,1/2)}\left(\frac{x}{l}\right)\left(1-\frac{x^2}{l^2}\right)^{1/2} \quad \text{for } -l\leqslant x\leqslant l,\ y=0, \tag{36}$$

$$\phi(x,0,t)=0 \quad \text{for } |x|>l,\ y=0, \tag{37}$$

where a_n and b_n are unknown coefficients to be determined and $P_n^{(1/2,1/2)}(x)$ is a Jacobi polynomial [13]. The Fourier transformation of Eqs. (34) and (36) is [4]

$$A(s)=\bar{w}(s,0,t)=\sum_{n=1}^{\infty} a_n B_n \frac{1}{s} J_{2n-1}(sl), \tag{38}$$

$$B(s)=\overline{\phi}(s,0,t)-\frac{e_{15}}{\varepsilon_{11}}\bar{w}(s,0,t)=\sum_{n=1}^{\infty}\left(b_n-\frac{e_{15}}{\varepsilon_{11}}a_n\right)B_n\frac{1}{s}J_{2n-1}(sl), \tag{39}$$

$$B_n=2\sqrt{\pi}(-1)^{n-1}\frac{\Gamma(2n-(1/2))}{(2n-2)!}, \tag{40}$$

where $\Gamma(x)$ and $J_n(x)$ are the Gamma and Bessel functions, respectively.

Substituting Eqs. (38) and (39) into Eqs. (27)–(30), respectively, Eqs. (28) and (30) can be automatically satisfied, respectively. Then the remaining equations (27) and (29) reduce to the form, respectively.

$$\sum_{n=1}^{\infty} a_n B_n \int_0^{\infty} \bar{\alpha}_0(s)\frac{\gamma}{s} J_{2n-1}(sl)\cos(sx)\,\mathrm{d}s=\frac{\pi}{2\mu}\tau_0(1+\lambda), \tag{41}$$

$$\sum_{n=1}^{\infty}\left(b_n-\frac{e_{15}}{\varepsilon_{11}}a_n\right)B_n\int_0^{\infty}\bar{\alpha}_0(s)J_{2n-1}(sl)\cos(sx)\,\mathrm{d}s=-\frac{\pi D_0}{2\varepsilon_{11}}, \tag{42}$$

where $\lambda=(e_{15}D_0)/\varepsilon_{11}\tau_0$. For large s, the integrands of Eqs. (41) and (42) almost decrease exponentially. So that they can be evaluated numerically by Filon's method (see e.g. [1]). Eqs. (41) and (42) can now be solved for the coefficients a_n and b_n by the Schmidt method [17]. For $a=0$, then $\bar{\alpha}_0(s)=1$ and Eqs. (41) and (42) reduce to the form of the same problem in classical piezoelectric materials. For brevity, Eq. (41) can be rewritten as (Eq. (42) can be solved by using a similar method):

$$\sum_{n=1}^{\infty} a_n E_n(x)=U(x), \quad -l<x<l, \tag{43}$$

where $E_n(x)$ and $U(x)$ are known functions and coefficients a_n are to be determined. A set of functions $P_n(x)$ which satisfy the orthogonality condition

$$\int_{-l}^{l} P_m(x)P_n(x)\,\mathrm{d}x=N_n\delta_{mn}, \quad N_n=\int_{-l}^{l} P_n^2(x)\,\mathrm{d}x \tag{44}$$

can be constructed from the function, $E_n(x)$, such that

$$P_n(x) = \sum_{i=1}^{n} \frac{M_{in}}{M_{nn}} E_i(x), \tag{45}$$

where M_{ij} is the cofactor of the element d_{ij} of D_n, which is defined as

$$D_n = \begin{bmatrix} d_{11}, d_{12}, d_{13}, \ldots, d_{1n} \\ d_{21}, d_{22}, d_{23}, \ldots, d_{2n} \\ d_{31}, d_{32}, d_{33}, \ldots, d_{3n} \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots\cdots\cdots \\ d_{n1}, d_{n2}, d_{n3}, \ldots, d_{nn} \end{bmatrix}, \quad d_{ij} = \int_{-l}^{l} E_i(x) E_j(x)\,\mathrm{d}x. \tag{46}$$

Using Eqs. (43)–(46), we obtain

$$a_n = \sum_{j=n}^{\infty} q_j \frac{M_{nj}}{M_{jj}} \quad \text{with } q_j = \frac{1}{N_j} \int_{-l}^{l} U(x) P_j(x)\,\mathrm{d}x. \tag{47}$$

5. Numerical calculations and discussion

From the references (see e.g. [15,16,35,36]), it can be seen that the Schmidt method is performed satisfactorily if the first ten terms of infinite series to Eq. (43) are retained. The behavior of the maximum dynamic stress stays steady with the increasing number in terms in Eq. (43). Although we can determine the entire dynamic stress field and the electric displacement from coefficients a_n and b_n, it is important in fracture mechanics to determine the dynamic stress τ_{yz} and the electric displacement D_y in the vicinity of the crack tips. τ_{yz} and D_y along the crack line can be expressed respectively as

$$\tau_{yz}(x,0,t) = -\frac{2}{\pi} \sum_{n=1}^{\infty} \left[\mu a_n B_n \int_0^{\infty} \bar{\alpha}_0(s) \frac{\gamma}{s} J_{2n-1}(sl) \cos(xs)\,\mathrm{d}s \right.$$
$$\left. + e_{15}\left(b_n - \frac{e_{15}}{\varepsilon_{11}} a_n \right) B_n \int_0^{\infty} \bar{\alpha}_0(s) J_{2n-1}(sl) \cos(xs)\,\mathrm{d}s \right], \tag{48}$$

$$D_y(x,0,t) = -\frac{2}{\pi} \sum_{n=1}^{\infty} (e_{15} a_n - \varepsilon_{11} b_n) B_n \int_0^{\infty} \bar{\alpha}_0(s) J_{2n-1}(sl) \cos(xs)\,\mathrm{d}s. \tag{49}$$

For $a = 0$ at $x = l$, we have the classical stress and electric displacement singularity. However, so long as $a \neq 0$, the semi-infinite integration and the series in Eqs. (48) and (49) are convergent for any variable x. Eqs. (48) and (49) give a finite stress all along $y = 0$, so there is no stress and electric displacement singularity at the crack tips. At $-l < x < l$, τ_{yz}/τ_0 and D_y/D_0 are very close to unity, and for $x > l$, τ_{yz}/τ_0 and D_y/D_0 possess finite values diminishing from a finite value at $x = l$ to zero at $x = \infty$. Since $a/2\beta l > 1/100$ represents a crack length of less than 100 atomic distances as stated by Eringen [9], and such submicroscopic sizes other serious questions arise regarding the interatomic arrangements and force laws, we do not pursue solutions valid at such small crack sizes. The semi-infinite numerical integrals, which occur, are evaluated easily by Filon's method (see e.g. [1]) and Simpson methods because of the rapid diminution of the integrands. In all computation, the material constants are not considered except the incident wave frequency, the wave velocity, the crack length and the lattice parameter in this paper. This is because the stress fields do not depend on the material constants. Due to the complexity of the integrands of Eqs. (48) and (49), the stress along the crack face has a slight variation. The results are plotted in Figs. 2–13. In Figs. 8–10, 12 and 13, $\tau_{yz}, \sigma_{yz}, D_y$ and D_y^{L} express the non-local stress, the local stress, the non-local electric displacement and the local electric displacement, respectively.

The following observations are very significant:

(i) The maximum stress does not occur at the crack tip, but slightly away from it. This phenomenon has been thoroughly substantiated by Eringen [11]. The maximum stress is finite. The distance between the crack tip and the maximum stress point is very small, and it depends on the crack length and the lattice parameter. Contrary to the classical piezoelectric theory solution, it is found that no stress and electric displacement singularity is present at the crack tip, and the present results converge to the classical ones at the points far away from the crack tip as shown in Figs. 8–13.

(ii) The dynamic stress and the electric displacement at the crack tip become infinite as the atomic distance $a \to 0$. This is the classical continuum limit of square root singularity.

(iii) For the $a/\beta =$ constant, viz., the atomic distance does not change, the value of the stress and the electric displacement concentrations (at the crack tip) increase with the increase of the crack length. Noting this fact, experiments indicate that the piezoelectric materials with smaller cracks are more resistant to fracture than those with larger cracks.

(iv) The significance of this result is that the fracture criteria are unified at both the macroscopic and microscopic scales, viz., it may solve the problem of any scale cracks.

(v) The present results will revert to the classical ones when the introduction function $\alpha(|X' - X|) = \delta(|X' - X|)$.

(vi) The dynamic stress concentration occurs at the crack tip as stated by Eringen [8,9], and this is given by

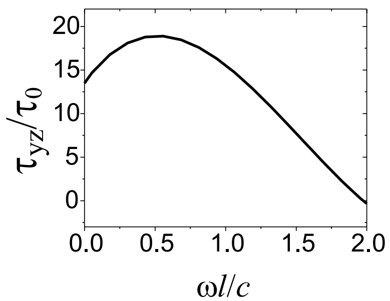

Fig. 2. The variation with $\omega l/c$ of the stress at the crack tips for $a/2\beta l = 0.001$.

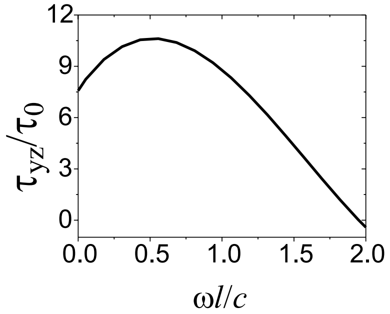

Fig. 3. The variation with $\omega l/c$ of the stress at the crack tips for $a/2\beta l = 0.003$.

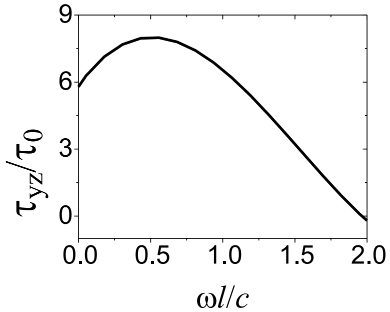

Fig. 4. The variation with $\omega l/c$ of the stress at the crack tips for $a/2\beta l = 0.005$.

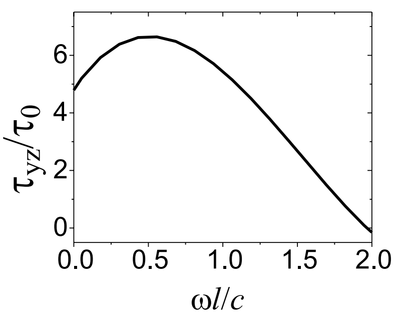

Fig. 5. The variation with $\omega l/c$ of the stress at the crack tips for $a/2\beta l = 0.007$.

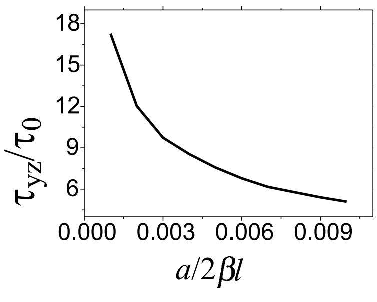

Fig. 6. The variation with $a/2\beta l$ of the stress at the crack tips for $\omega l/c = 1.0$.

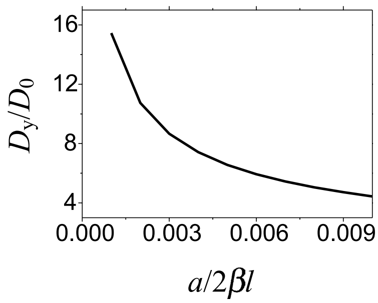

Fig. 7. The variation with $a/2\beta l$ of the electric displacement at the crack tips.

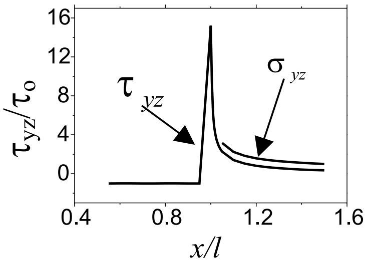

Fig. 8. The variation of the stress along the crack line for $a/2\beta l = 0.001$, $\omega l/c = 0.0$.

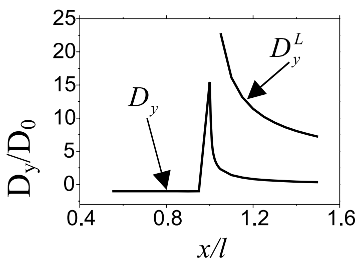

Fig. 9. The variation of the elastic displacement along the crack line for $a/2\beta l = 0.001$.

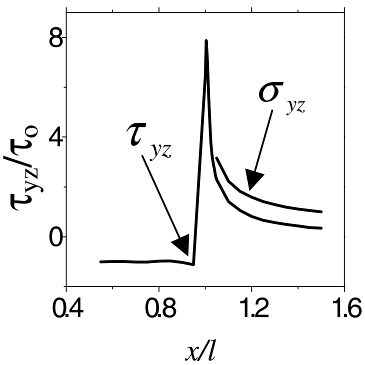

Fig. 10. The variation of the stress along the crack line for $a/2\beta l = 0.005$, $\omega l/c = 0.0$.

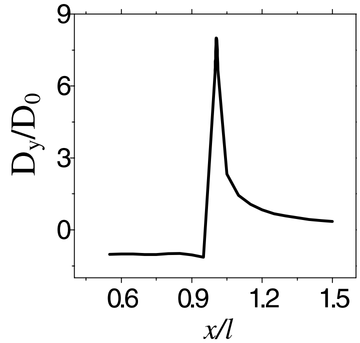

Fig. 11. The variation of the elastic displacement along the crack line for $a/2\beta l = 0.005$.

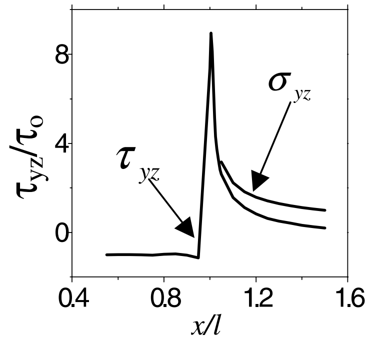

Fig. 12. The variation of the stress along the crack line for $a/2\beta l = 0.005$, $\omega l/c = 1.0$.

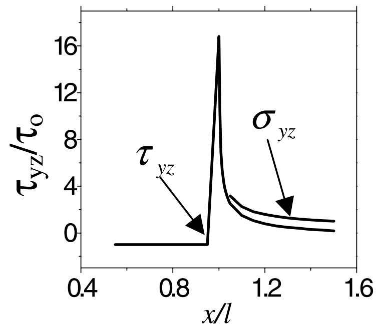

Fig. 13. The variation of the stress along the crack line for $a/2\beta l = 0.001$, $\omega l/c = 1.0$.

$$\tau_{yz}(l, 0, t)/\tau_0 = c_0/\sqrt{a/2\beta l}, \tag{50}$$

where c_0 represents the stress concentration value at tip of the crack. The c_0 is about equal to $c_0 \approx 0.533$. It is larger than the static stress concentration of the static non-local problem [9]. (vii) The dimensionless stress is found to be independent of the electric loads and the material parameters. It just depends on the length of the crack, the lattice parameter, the circular

frequency of the incident wave and the wave velocity. However, the electric field is found to be independent of the material parameters and the circular frequency of the incident wave and the wave velocity. It just depends on the length of the crack, the lattice parameter.
(viii) The dynamic stress at the crack tips tends to increase with the frequency research a peak and then to decrease in magnitude.
(ix) The dynamic stress and the dynamic electric displacement at the crack tips tend to decrease with increasing $a/2\beta l$.

6. Conclusions

We developed an electro-elastic fracture mechanics theory and the non-local theory to determine the stress and electric fields near the crack tip for piezoelectric materials having a Griffith crack under dynamic loading. The anti-plane electro-elastic problem of the piezoelectric materials with a crack has been analyzed theoretically. The traditional concept of linear elastic fracture mechanics and the non-local theory is extended to include the piezoelectric effects and the results are expressed in terms of the stress and electric fields. The development method is applied to illustrate the fundamental behavior of a crack in piezoelectric materials under dynamic loading. Furthermore, the shear stress wave velocity of the piezoelectric materials and the frequency of the incident wave upon the dynamic stress field of the crack are examined.

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